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DETERMINATION OF NORMAL FORCES ARISING FROM IN-BORE PRESSURES ON AN N-SEGMENTED SABOT: SINGLE FLECHETTE

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A theoretical model for determining the normal forces produced on an n-segmented sabot during the interior ballistic cycle was developed. While the model considers frustrums of cones and includes the cylinder as a special case, the vector methods employed can be extended to any shape. This model assumes that the sabot can be approximated by segmented rigid bodies positioned symmetrically and individually spaced about the projectile. Results indicate that the composite normal force increases directly with the number $(n \geq 2)$ of segments and approaches a limiting value as n approaches infinity. Furthermore, it is shown that sixteen segments approximates this limit to within 0.7 percent. Knowledge of the normal force can be used to develop conditions for preventing sabot/projectile slipping, as well as establishing material requirements.

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A theoretical model for determining the normal forces produced on an n-segmented sabot during the interior ballistic cycle was developed. While the model considers frustrums of cones and includes the cylinder as a special case, the vector methods employed can be extended to any shape. This model assumes that the sabot can be approximated by segmented rigid bodies positioned symmetrically and individually spaced about the projectile. Results indicate that the composite normal force increases directly with the number ($n \geq 2$) of segments and approaches a limiting value as n approaches infinity. Furthermore, it is shown that sixteen segments approximates this limit to within 0.7 percent. Knowledge of the normal force can be used to develop conditions for preventing sabot/projectile slipping, as well as establishing material requirements.

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INTRODUCTION

In the quest for improved weapon systems and firepower, recent trends have been toward saboted projectiles, flechettes in particular. The sabot serves as a carrier for a projectile or flechette whose caliber is usually much smaller than the bore diameter. In addition to serving as the carrier, the sabot acts as an obturator thus preventing the gases from escaping. The forces that will be acting on it must be known for adequate and optimal structural design, selection of materials, and determination of relevant sabot/projectile interface parameters. These forces arise primarily from in-bore pressures, compression from the bore, and aerodynamic forces when the sabot exits, see Figure 1.

This report is concerned only with the forces (Region I) resulting from the propellant gases during the interior ballistic phase. With the assumption that the sabot consists of segments approximating rigid bodies, these forces are resolved into radial or normal and axial components. The geometry of the sabots to be considered consist of n-axially symmetric segments individually spaced and infinitestimally separated about the projectile. This geometry includes the cylinder as a special case. In addition, the flechette is assumed to have an unserrated cylindrical body where it makes contact with the sabot. The vector methods, however, are generally applicable and other geometries can be investigated following a similar analysis. The value of determining the composite normal and axial forces cannot be overemphasized. They determine the anticipated loads required of the sabot material and provide a method for determining frictional requirements for preventing sabot/projectile slips. The latter problem is subsequently discussed.

THEORY

The sabot under consideration in this study has a tail section in the form of a frustrum having length, ℓ, and radii, r and R, see Figure 2A. The outer surface of the frustrum is given by:

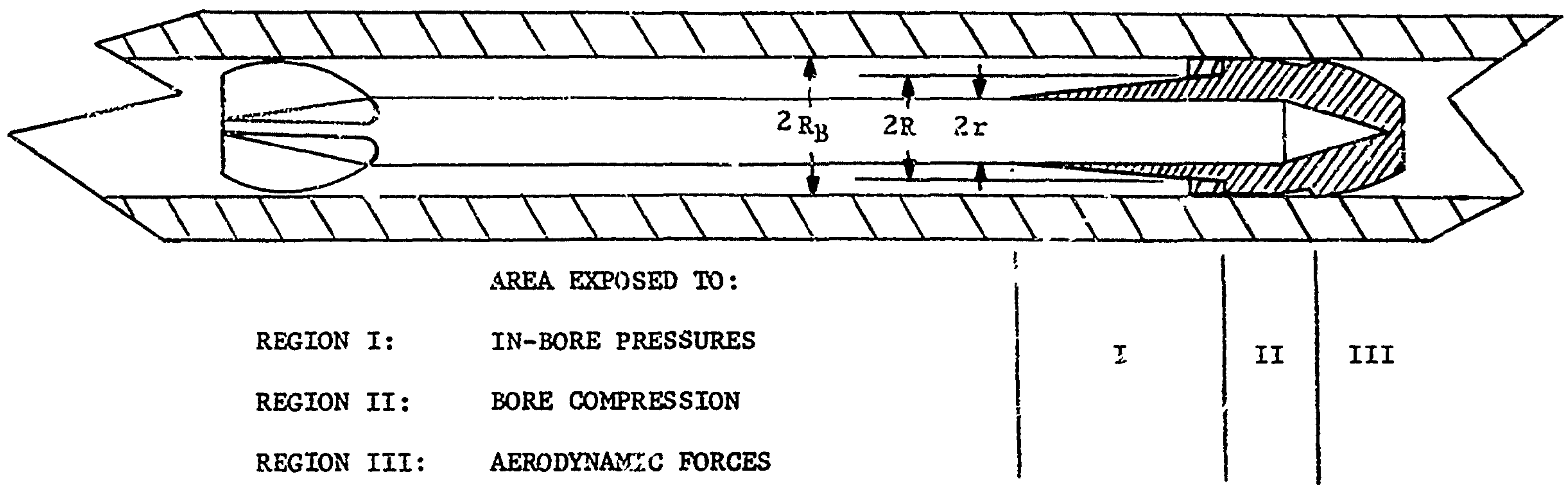

Figure 1. Typical Saboted Flechette Round

$$S(x, y, z) = (\ell_0 + \ell)^2 (x^2 + y^2) - R^2 z^2 = 0 \tag{1}$$

where

$$\ell_0 \leq z \leq \ell_0 + \ell$$

and

$$r = R \ell_0 / (\ell_0 + \ell)$$

During the interior ballistic cycle, the propellant gases will exert a resultant force, $\overline{F}$, on the sabot. This force is calculated from:

$$\overline{F} = - \oiint_S P \, d\overline{A} \tag{2}$$

when P is the pressure of the gases and the integral is evaluated over the sloped surface of the frustrum. The k component of this force causes the sabot to accelerate down the gun barrel, while the other components combine to compress the sabot against the flechette. Compression gives rise to a frictional force, f , between the sabot and the flechette which can be expressed as:

$$f = \mu N \tag{3}$$

where μ is the coefficient of friction, and N the normal force keeping the two bodies in contact. For a truly rigid body sabot, the resultant normal force is zero and, consequently, there is no frictional force. However, for a sabot composed of n individual segments, each free to move, there is a resultant normal force. For this model, each segment consists of a wedge of angle $\theta = 2\pi/n$, see Figure 2B. The resultant force on a given wedge is:

$$\overline{F} = - \oiint_{S'} P \hat{n} \, dA \tag{4}$$

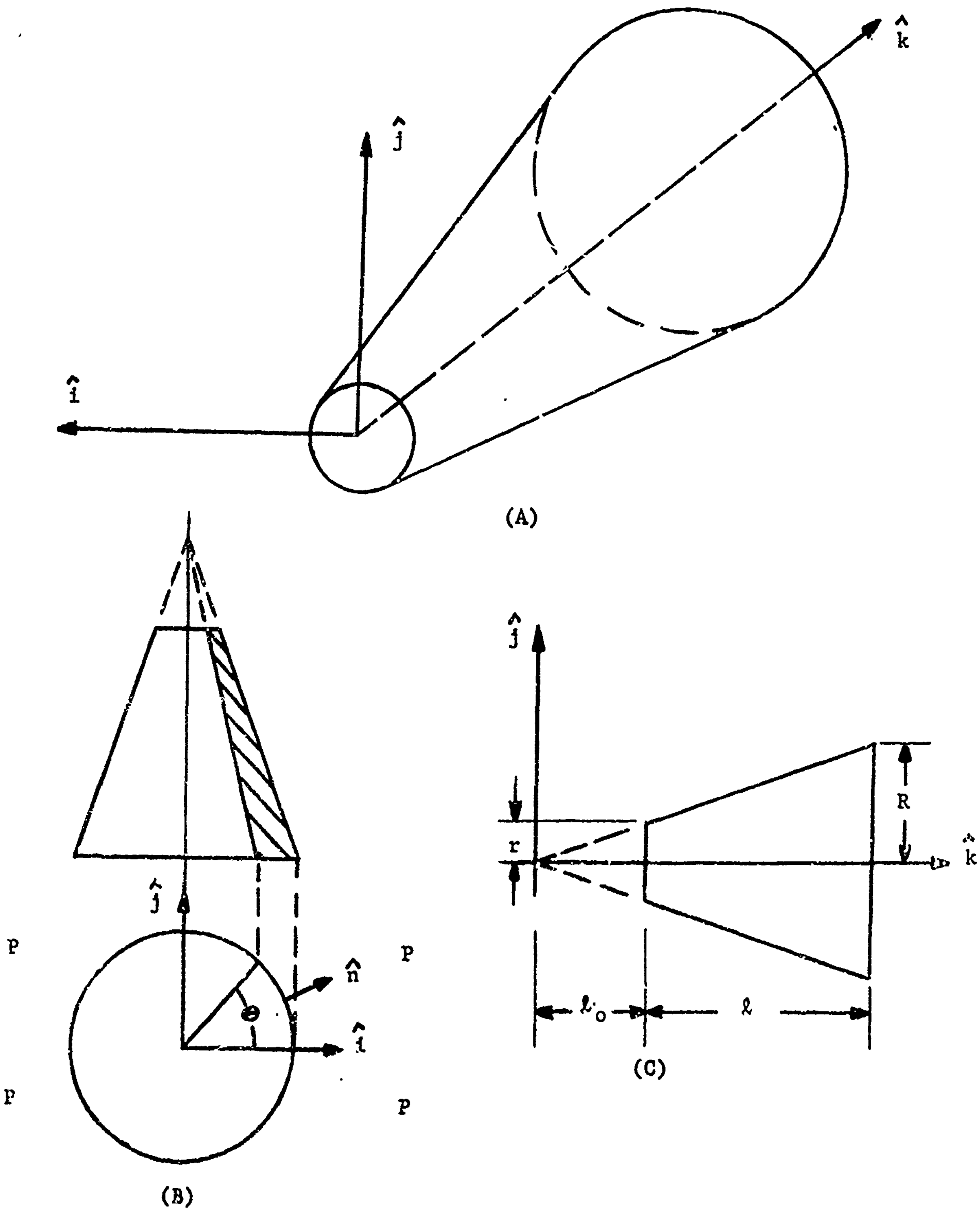

Figure 2. Sabot Geometry

where the integral is evaluated over the curved surface S' of the wedge and $\hat{n}$ is the outward normal to this surface. Using a theorem from vector analysis[1,2] and choosing the coordinate axes appropriately, Equation 4 becomes:

$$\overline{F} = - \int_{\ell_o}^{\ell_o + \ell} dz \int_{Rz\cos\theta/(\ell_o+\ell)}^{Rz/(\ell_o+\ell)} P\hat{n} \frac{dx}{|\hat{n}\cdot\hat{j}|} \tag{5}$$

The outward normal $\hat{n}$ is found from Equation 1:

$$\hat{n} = \frac{\overline{\nabla} S}{|\overline{\nabla} S|} = \frac{(\ell_o + \ell)^2 (x\hat{i} + y\hat{j}) - R^2 z\hat{k}}{zR\left[(\ell_o + \ell)^2 + R^2\right]^{1/2}} \tag{6}$$

and:

$$|\hat{n}\cdot\hat{j}| = \frac{(\ell_o + \ell)^2 y}{zR\left[(\ell_o + \ell)^2 + R^2\right]^{1/2}}$$

Assuming that the pressure is uniform over the surface of the wedge and varies only in time, one obtains the i^{th} component of the force:

$$F_i = P(t) \int_{\ell_o}^{\ell_o + \ell} dz \int_{Rz\cos\theta/(\ell_o+\ell)}^{Rz/(\ell_o+\ell)} dx \frac{x}{\left[\frac{R^2 z^2}{(\ell_o+\ell)^2} - x^2\right]^{1/2}}$$

$$= - P(t) \sin\theta \frac{R}{2} \frac{\left[(\ell_o + \ell)^2 - \ell_o^2\right]}{\ell_o + \ell} \tag{7}$$

[1] Wilfred Kaplan, "Advanced Calculus," Addison-Wesley, Massachusettes, 1952, pp 262-267.

[2] Murray R. Spiegal, "Vector Analysis," Schaum Publishing Co., New York, 1959, pp 94-95.

Given $R\ell_o/(\ell_o + \ell) = r$, this becomes:

$$F_i = - P(t) \sin\theta \left(\frac{R+r}{2}\right) \ell \qquad (8)$$

Similarly, for the j component of the force:

$$F_j = - P(t) \int_{\ell_o}^{\ell_o+\ell} dz \int_{Rz\cos\theta/(\ell_o+\ell)}^{Rz/(\ell_o+\ell)} dx$$

$$= - P(t)\ (1 - \cos\theta) \left(\frac{R+r}{2}\right) \ell \qquad (9)$$

Finally, the force in the k direction is:

$$F_k = P(t) \int_{\ell_o}^{\ell_o+\ell} zdz \int_{Rz\cos\theta/(\ell_o+\ell)}^{Rz/(\ell_o+\ell)} dx\ \frac{R^2}{(\ell_o+\ell)^2}\ \frac{x}{\left[\dfrac{R^2 z^2}{(\ell_o+\ell)^2} - x^2\right]^{1/2}}$$

$$= P(t) \left(\frac{R^2 - r^2}{2}\right) \theta \qquad (10)$$

Thus, the total force acting on the wedge is:

$$\overline{F} = - P(t) \left(\frac{R + r}{2}\right) \ell \left[\sin\theta\, \hat{i} + (1 - \cos\theta)\, \hat{j}\right]$$

$$+ P(t) \left(\frac{R^2 - r^2}{2}\right) \theta\, \hat{k} \qquad (11)$$

From this equation, one finds that the normal force has magnitude N:

$$N = \left[F_i^2 + F_j^2\right]^{1/2}$$

$$= P(t)\ (R + r)\ \ell\ \sin\frac{\theta}{2} \qquad (12)$$

and acts along the line which bisects the angle θ of the wedge and passes through the center of mass of the segment.

Since the problem possesses cylindrical symmetry, an arbitrary segment of the sabot will experience a force in the k direction equal to that in Equation 10 and a radial force with magnitude given by Equation 12 along the line bisecting that particular segment. Then, resolving the radial force into its i and j components, the resultant force acting on the m^{th} segment of the sabot becomes:

$$\overline{F}^{(m)} = - P(t)\,(R + r)\,\ell\,\sin\frac{\theta}{2}\left[\cos\left(m - \frac{1}{2}\right)\theta\,\hat{i}\right.$$

$$\left. + \sin\left(m - \frac{1}{2}\right)\theta\,\hat{j}\right] + P(t)\left(\frac{R^2 - r^2}{2}\right)\theta\,\hat{k} \qquad (13)$$

where the segments have been numbered counterclockwise and $m = 1, 2, \ldots n$.

Therefore, the k component of the total force acting on the frustrum is:

$$F_k^T = \sum_{m=1}^{n} F_k^{(m)} = n F_k = P(t)\,(R^2 - r^2)\,\pi \qquad (14)$$

which is simply the pressure of the gases times the projected area of the frustrum perpendicular to the k direction.

As each of the segments of the sabot exerts a normal force on the flechette, the composite normal force N_c is:

$$N_c = \sum_{m=1}^{n} N^{(m)} = nN = P(t)\,(R + r)\,\ell\, n \sin\frac{\pi}{n} \qquad (15)$$

This equation demonstrates that when the sabot is considered as a rigid body without any slits, the frictional force between it and the flechette is zero. Only when the sabot is comprised of segments is a frictional force generated, and this force increases as the number of segments is increased. In reality, there are no rigid bodies and the shape of the sabot will yield under stress. Consequently, even for a sabot with no slits, there is some frictional force. However, for sabots made of materials which are rather inelastic, this force is quite small.

The equations derived can be applied to the case of a cylindrical tail section by simply letting $r = R$, $R < R_B$ in Equations 11, 12, 13, and 15. Should the sabot be composed of several different sections, the above analysis holds for each section and the total normal force is given by the sum of the composite normal forces.

RESULTS

Investigating the behavior of the composite normal force for large n reveals:

$$\lim_{n\to\infty} - P(t)(R+r)\,\ell\, n \sin\frac{\pi}{n} = - P(t)\,\ell\,(R+r) \lim_{n\to\infty} n \sin\frac{\pi}{n}$$

$$= - P(t)\,\ell\,(R+r) \lim_{n\to\infty} n \left[\frac{\pi}{n} - \frac{\pi^3}{3!\,n^3} + \frac{\pi^5}{5!\,n^5} - \cdots\right]$$

$$= - P(t)\,\ell\,(R+r)\,\pi$$

The following table indicates the rapidity of the convergence of $n \sin \pi/n$ to π as n increases.

TABLE I.
Convergence of $n \sin \pi/n$

n	$n \sin \pi/n$
2	2.0000
4	2.8284
8	3.0616
16	3.1216
.	.
.	.
.	.
∞	= 3.1416

Considering the percentage error in the normal force from using 16 segments instead of the limiting case yields

$$- \frac{P\ell\,(R+r)\,\pi - (\,- P\ell\,(R+r)\,(3.1216)}{- P\ell\,(R+r)\,\pi} \times 100\%$$

$$= \frac{3.1416 - 3.1216}{3.1416} \times 100\% = 0.7\%$$

Thus, for practical purposes increasing the number of segments beyond 16 to provide for a higher normal force becomes insignificant.

In addition to increasing the number of segments, variations to the normal force are caused by varying R, r, and ℓ individually or in combination. As long as the variation selected increases (decreases) the product ℓ (R + r) there will be an increase (decrease) to the normal force.

CONCLUSIONS

For segmented sabots increasing the number of segments increases the composite normal force which is an important consideration from the standpoint of the frictional requirements for preventing sabot/projectile slips. Consideration of the equations of motion of the sabot and the projectile coupled with the requirement for zero acceleration between them yields a lower limit for the frictional force required. Assuming this force is simply proportional to the normal force fixes a restriction on the product of the coefficient of friction and the normal force. Since this coefficient varies inversely with the normal force, a realistic benefit can be realized by increasing the number of segments up to 16. If this fails, other sabot materials (compatible with stress requirements) can be selected or various types of coatings may be investigated to improve the frictional coefficient between the sabot and the flechette. Finally, slightly oversized (diameter) ribs can be added to the sabot. This will transform the cross-sectional area from a circular to a crucifix form and will provide an additional compressive force and improved projectile alignment.